EE5106R: Advanced Robotics Part II Quiz Solution

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a) $$\begin{bmatrix} M_1 + 2M_2c_2 & \frac{2}{3}M_2 + M_2c_2 \\ \frac{2}{3}M_2 + M_2c_2 & \frac{2}{3}M_2 \end{bmatrix} a = \begin{bmatrix} (M_1 + 2M_2c_2)a_1 + \left(\frac{2}{3}M_2 + M_2c_2\right)a_2 \\ \left(\frac{2}{3}M_2 + M_2c_2\right)a_1 + \left(\frac{2}{3}M_2\right)a_2 \end{bmatrix}$$

$$\begin{bmatrix} -M_2s_2\dot{q}_2 & -M_2s_2(\dot{q}_1 + \dot{q}_2) \\ M_2s_2\dot{q}_1 & 0 \end{bmatrix} v = \begin{bmatrix} -M_2s_2\dot{q}_2v_1 - M_2s_2(\dot{q}_1 + \dot{q}_2)v_2 \\ M_2s_2\dot{q}_1v_1 \end{bmatrix}$$

Then, we have

$$\tau = \begin{bmatrix} (M_1 + 2M_2c_2)a_1 + \left(\frac{2}{3}M_2 + M_2c_2\right)a_2 - M_2s_2\dot{q}_2v_1 - M_2s_2(\dot{q}_1 + \dot{q}_2)v_2 + M_3c_1 + M_4c_{12} + 2M_4c_1 \\ \left(\frac{2}{3}M_2 + M_2c_2\right)a_1 + \left(\frac{2}{3}M_2\right)a_2 + M_2s_2\dot{q}_1v_1 + M_4c_{12} \end{bmatrix}$$
$$= \begin{bmatrix} a_1 & 2c_2a_1 + \left(\frac{2}{3} + c_2\right)a_2 - s_2\dot{q}_2v_1 - s_2(\dot{q}_1 + \dot{q}_2)v_2 & c_1 & c_{12} + 2c_1 \\ 0 & \left(\frac{2}{3} + c_2\right)a_1 + \frac{2}{3}a_2 + s_2\dot{q}_1v_1 & 0 & c_{12} \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix}$$

Therefore we get

$$\Psi(q, \dot{q}, v, a) = \begin{bmatrix} a_1 & 2c_2a_1 + \left(\frac{2}{3} + c_2\right)a_2 - s_2\dot{q}_2v_1 - s_2(\dot{q}_1 + \dot{q}_2)v_2 & c_1 & c_{12} + 2c_1 \\ 0 & \left(\frac{2}{3} + c_2\right)a_1 + \frac{2}{3}a_2 + s_2\dot{q}_1v_1 & 0 & c_{12} \end{bmatrix}$$

b) There are two approaches to obtain the dynamics of the robot. First is the energy based method – Lagrange-Euler formulation, and second is the momentum/force approach – Newton-Euler formulation. In general, both derivations give similar final results. The approaches, however, are different.

In Newton-Euler formulation, the angular velocity, angular acceleration, linear velocity and linear acceleration of each link is computed in terms of its preceding link. Therefore these values can be computed in a recursive manner starting from the base to the end-effector link. This method is efficient, and the derivations are simple. The computation is, however, messy especially at complicated robot manipulators.

For Lagrange-Euler formulation, it only requires the calculation of the total kinetic energy and potential energy of a manipulator, and thus forms the Lagrangian equation. It is simple in the sense that it has a closed form solution, in the form of $D\ddot{q} + C\dot{q} + G = \tau$ where D, C, G can all be found following a few simple steps.

In the presence of complicated manipulator, Lagrange-Euler formulation will be preferred due to its simplicity and having closed-form solution. In the case where simple manipulator is used, Newton-Euler formulation provides more information of each joints and it allows real time control of the manipulator.