EE5103 - Computer Control Systems
Homework #3

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1. (a) Transfer function is given by

$$H(z) = \frac{z+0.8}{z^2-2z+1} \equiv \frac{B(z)}{A(z)}$$

Closed-loop characteristic polynomial is then given by

$$\begin{aligned} A_{cl} &= A(z)R(z) + B(z)S(z) \\ &= (z^2-2z+1)R(z) + (z+0.8)S(z) \end{aligned}$$

Since the process zero is stable ($z = -0.8$), we can design $R(z)$ such that it cancel the process zero, as follows:

$$A_{cl} = (z^2-2z+1)(z+0.8) + (z+0.8)S(z)$$

Letting $S(z) = s_0 z + s_1$, we have

$$\begin{aligned} A_{cl} &= (z+0.8)(z^2-2z+1+s_0 z+s_1) \\ &= (z+0.8)(z^2+(s_0-2)z+(s_1+1)) \\ &\equiv (z+0.8)A_m(z) \end{aligned}$$

By comparing coefficient, we get

$$\begin{aligned} s_0 &= 0.2 \\ s_1 &= -0.1 \end{aligned}$$

Thus, the closed-loop transfer function becomes

$$H_{cl}(z) = \frac{T(z)}{z^2-1.8z+0.9}$$

To obtain steady-state gain 1, we compute $H_{cl}(1) = T(1)/0.1 = 1$. Therefore, $T(z) = 0.1$. Therefore, the controller is then given by

$$\begin{aligned} (q+0.8)u(k) &= 0.1u_c(k) - (0.2q-0.1)y(k) \\ u(k+1) + 0.8u(k) &= 0.1u_c(k) - 0.2y(k+1) + 0.1y(k) \\ u(k) &= -0.8u(k-1) + 0.1u_c(k-1) - 0.2y(k) + 0.1y(k-1) \end{aligned}$$

(b) Similarly,

$$A_{cl} = (z^2-2z+1)R(z) + (z+0.8)S(z)$$

This time, however, the process zero can not be canceled. One can observe that there will be insufficient equations to obtain the required $A_m(z)$. Thus, by letting $A_0(z) = z$, we have

$$\begin{aligned} A_{cl} &= A_0(z)A_m(z) \\ &= z^3 - 1.8z^2 + 0.9z \\ &\equiv (z^2 - 2z + 1)R(z) + (z + 0.8)S(z) \end{aligned}$$

Letting $R(z) = z + r_1$, and $S(z) = s_0 z + s_1$, we have

$$\begin{aligned} A_{cl} &= (z^2 - 2z + 1)(z + r_1) + (z + 0.8)(s_0 z + s_1) \\ &= z^3 + (r_1 + s_0 - 2)z^2 + (1 - 2r_1 + 0.8s_0 + s_1)z + (r_1 + 0.8s_1) \end{aligned}$$

By comparing coefficient, we have

$$\begin{aligned} S(z) &= \frac{11}{81}z - \frac{13}{162} \\ R(z) &= z + \frac{26}{405} \end{aligned}$$

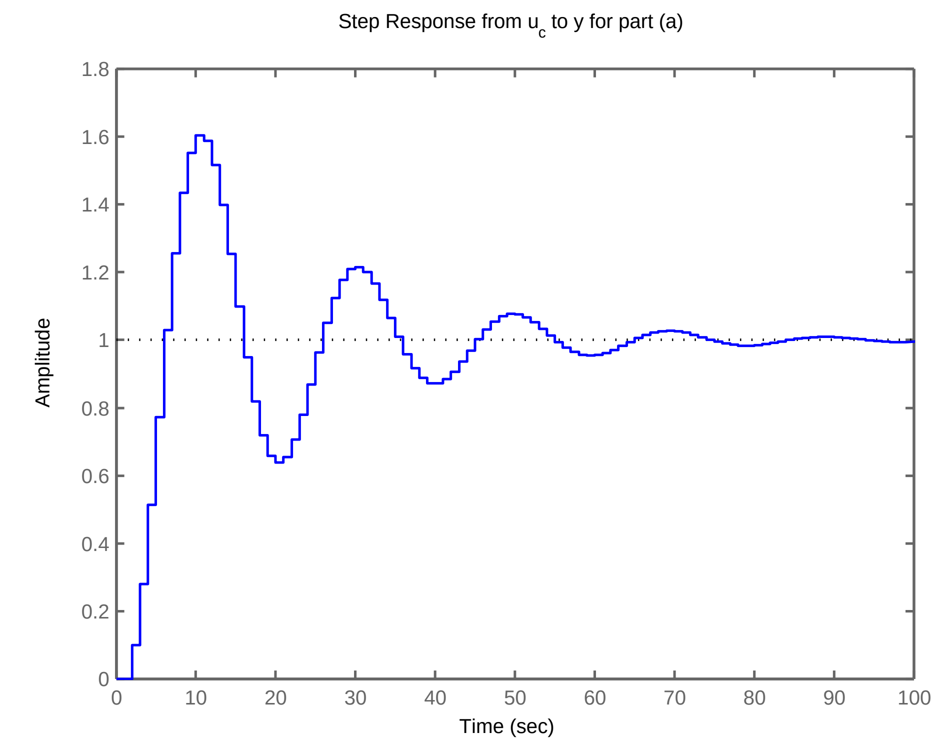
Now, $T(z) = t_0 A_0(z) = t_0 z$. To make steady-state gain 1, we need $T(z) = \frac{1}{18}z$. The closed-loop transfer function is thus

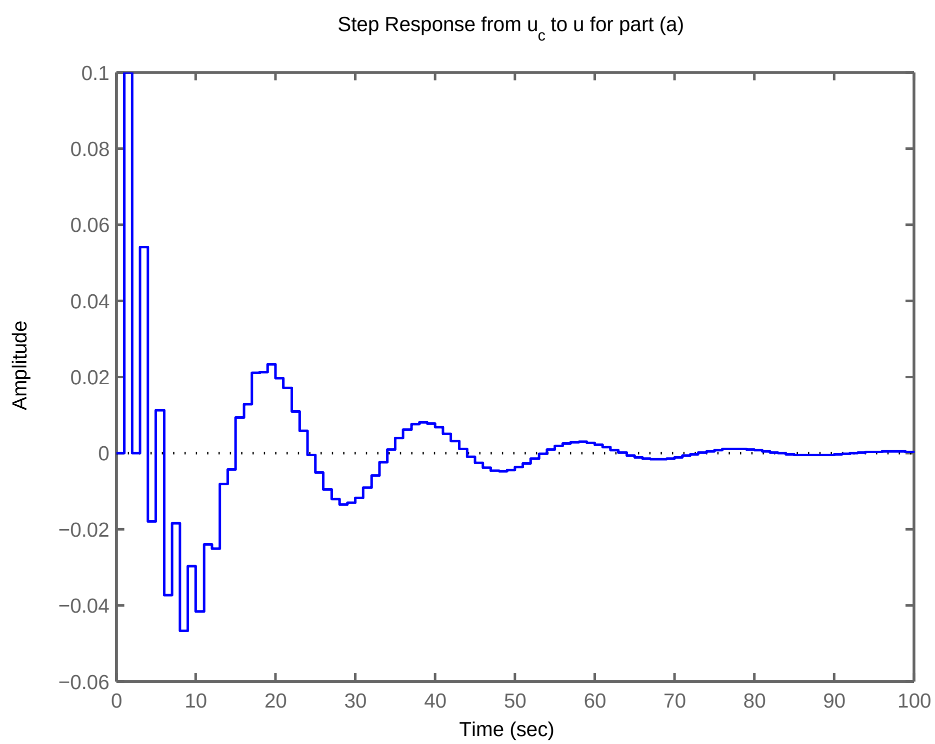
$$H_{cl}(z) = \frac{z + 0.8}{18(z^2 - 1.8z + 0.9)}$$

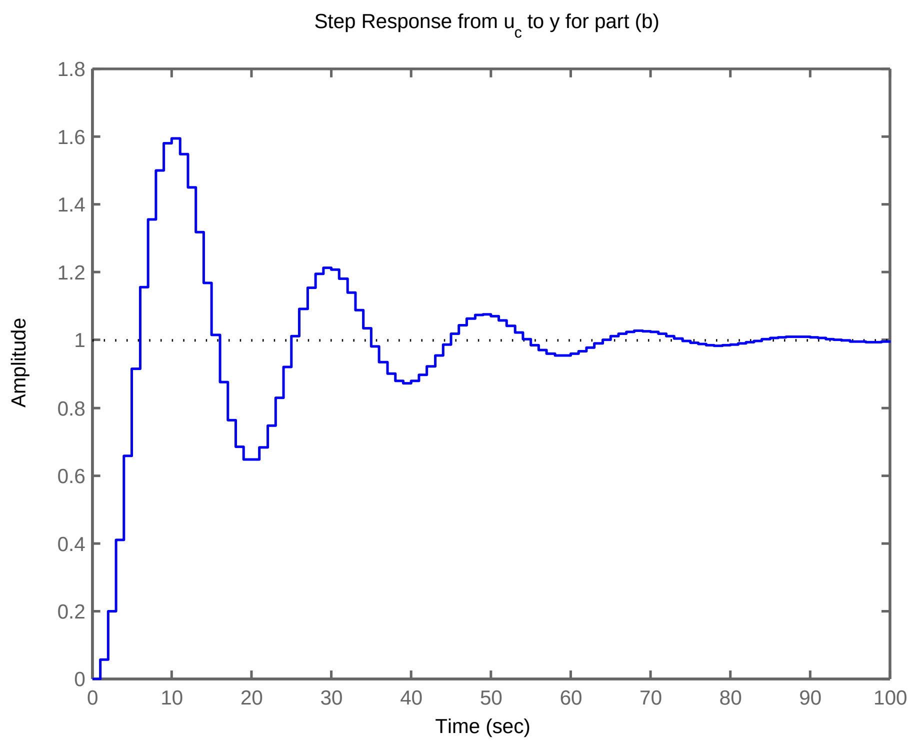
The controller is then

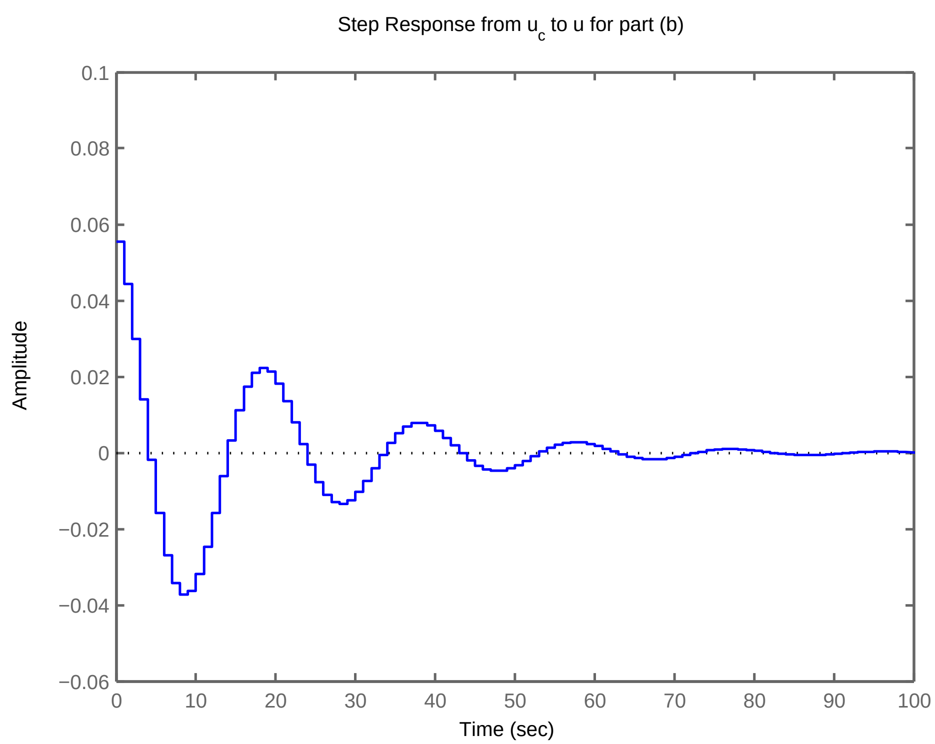
$$\begin{aligned} (q + 0.0642)u(k) &= 0.0556qu_c(k) - (0.1358q - 0.0802)y(k) \\ u(k+1) + 0.0642u(k) &= 0.0556u_c(k+1) - 0.1358y(k+1) + 0.0802y(k) \\ u(k) &= -0.0642u(k-1) + 0.0556u_c(k) - 0.1358y(k) + 0.0802y(k-1) \end{aligned}$$

(c) The following plots show the output and input signals for both cases in step response. We can observe that the output of both the system behaved almost similarly. However, the input signal of the first case (where process zero was canceled) has huge oscillation at the beginning. Therefore in my opinion, the case 2 (where process zero was not canceled) is preferred.


Step Response from u_c to y for part (a)
Amplitude
Time (sec)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
100


Step Response from u_c to u for part (a)
Amplitude
Time (sec)
0.1
0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
0
10
20
30
40
50
60
70
80
90
100


Step Response from u_c to y for part (b)
Amplitude
Time (sec)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
100


Step Response from u_c to u for part (b)
Amplitude
Time (sec)
0.1
0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
0
10
20
30
40
50
60
70
80
90
100

2. (a) Express the system in the transfer function form, we have

$$\begin{aligned}
Y(z) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z-0.5 & -1 \\ -0.2 & z-0.8 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} U(z) + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z-0.5 & -1 \\ -0.2 & z-0.8 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} V(z) \\
&= \frac{z+0.2}{z^2-1.3z+0.2} U(z) + \frac{z-0.8}{z^2-1.3z+0.2} V(z) \\
&\equiv \frac{B_1(z)}{A(z)} U(z) + \frac{B_2(z)}{A(z)} V(z)
\end{aligned}$$

Now, we design the controller in the form of

$$R(q)u(k) = T(q)u_c(k) - S(q)y(k)$$

Then,

$$Y(z) = \frac{B_1(z)T(z)}{A(z)R(z)+B(z)S(z)} U_c(z) + \frac{B_2(z)R(z)}{A(z)R(z)+B(z)S(z)} V(z)$$

Since disturbance $v(k)$ is a constant, we need to design $R(z)$ such that it contain $(z-1)$ term. Let $R(z) = (z-1)R'(z)$, we have

$$A_{cl} = (z^2-1.3z+0.2)(z-1)R'(z) + (z+0.2)S(z)$$

Next, we design $R'(z) = z + r_1$ and $S(z) = s_0 z^2 + s_1 z + s_2$, we have

$$\begin{aligned}
A_{cl} &= (z^2-1.3z+0.2)(z-1)(z+r_1) + (z+0.2)(s_0 z^2 + s_1 z + s_2) \\
&= z^4 + (r_1 + s_0 - 2.3)z^3 + (1.5 - 2.3r_1 + s_1 + 0.2s_0)z^2 \\
&\quad +(1.5r_1 - 0.2 + s_2 + 0.2s_1)z + (0.2s_2 - 0.2r_1)
\end{aligned}$$

By designing all the closed-loop pole at $z = 0$, i.e., $A_m(z) = z^4$, we have

$$\begin{aligned}
s_2 &= 0.1973 \\
s_1 &= -1.4667 \\
s_0 &= 2.1027 \\
r_1 &= 0.1973
\end{aligned}$$

To assign the steady-state gain at 1, we need $H_1(1) = \frac{1+0.2}{1} t_0 = 1$. Thus, $t_0 = 0.8333$. Then

$$\begin{aligned}
\frac{Y(z)}{U_c(z)} &= \frac{0.8333(z+0.2)}{z^4} \\
\frac{Y(z)}{V(z)} &= \frac{z-0.8}{z^4}
\end{aligned}$$

By this design method, we can completely eliminate the process disturbance v. The controller is

$$\begin{aligned}
(q^2 - 0.8027z - 0.1973)u(k) &= 0.8333u_c(k) - (2.1027q^2 - 1.4667q + 0.1973)y(k) \\
u(k) &= 0.8027u(k-1) + 0.1973u(k-2) + 0.8333u_c(k-2) \\
&\quad -2.1027y(k) + 1.4667y(k-1) - 0.1973y(k-2)
\end{aligned}$$

(b) This design by using polynomial has lower computation intensity as compared to the Prob. 2 in Homework #2 as it does not require the matrices computation to be used in Achermann's formula, also, by using this method, we are not required to design the observer for the closed-loop system. Therefore this is a simpler solution.

3. (a) Similar to Prob. 1, the closed-loop characteristic polynomial is given by

$$\begin{aligned} A_{cl} &= A(z)R(z) + B(z)S(z) \\ &= (z^2 - 4z + 3)R(z) + (z - 0.5)S(z) \end{aligned}$$

Since the process zero is stable, we can make $R(z) = z - 0.5$ to cancel it. Also, letting $S(z) = s_0 z + s_1$, we have

$$\begin{aligned} A_{cl} &= (z - 0.5)(z^2 - 4z + 3 + s_0 z + s_1) \\ &= (z - 0.5)(z^2 + (s_0 - 4)z + (s_1 + 3)) \end{aligned}$$

We then have $s_0 = 4$ and $s_1 = -3$ to match the given $A_m(z) = z^2$. Thus, $T(z) = 1$ will perfectly match the reference model, as follows:

$$H_{cl} = \frac{z - 0.5}{z^2(z - 0.5)} = \frac{1}{z^2}$$

The controller is then

$$\begin{aligned} (q - 0.5)u(k) &= u_c(k) - (4q - 3)y(k) \\ u(k) &= 0.5u(k-1) + u_c(k-1) - 4y(k) + 3y(k-1) \end{aligned}$$

(b) We first design the feedback system. The transfer function of the feedback system is given by

$$H_{fb}(z) = \frac{R(z)B(z)}{A(z)R(z) + B(z)S(z)}$$

Here, we can assign the closed-loop pole location by choosing proper $R(z)$ and $S(z)$. Suppose we design the closed-loop system to be a deadbeat system, then $A_m = z^3$. Let $R(z) = z + r_1$, and $S(z) = s_0 z + s_1$.

$$\begin{aligned} A_{cl} &= (z^2 - 4z + 3)(z + r_1) + (z - 0.5)(s_0 z + s_1) \\ &= z^3 + (r_1 + s_0 - 4)z^2 + (3 + s_1 - 4r_1 - 0.5s_0)z + (3r_1 - 0.5s_1) \equiv z^3 \end{aligned}$$

Solving them, we have

$$\begin{aligned} r_1 &= -0.4 \\ s_0 &= 4.4 \\ s_1 &= -2.4 \end{aligned}$$

Next, we design the feedforward controller as follows

$$\begin{aligned} H_{ff}(z)H_{fb}(z) &= H_m(z) \\ H_{ff}(z)\frac{(z - 0.5)(z - 0.4)}{z^3} &= \frac{1}{z^2} \end{aligned}$$

As the zeroes are stable, we simply let $H_{ff}(z) = \frac{z}{(z-0.5)(z-0.4)}$ to satisfy the equation. Thus the overall controller will track the reference model perfectly. The controller is

$$\begin{aligned} U(z) &= U_{fb}(z) + U_{ff}(z) \\ &= -\frac{4.4z - 2.4}{z - 0.4}Y(z) + \frac{z}{(z - 0.5)(z - 0.4)}U_c(z) \end{aligned}$$

4. (a) By shifting the time step, we have

$$\begin{aligned} y(k+2) &= cy(k+1) + \sin(y(k)) + u(k) \\ &= c^2 y(k) + c\sin(y(k-1)) + cu(k-1) + \sin(y(k)) + u(k) \end{aligned}$$

Then, a one-step-ahead controller can be designed by letting

$$u(k) = y^*(k+2) - c^2 y(k) - c\sin(y(k-1)) - cu(k-1) - \sin(y(k))$$

such that $y(k+2) = y^*(k+2)$.

(b) We express the difference equation as

$$u(k) = y(k+2) - cy(k+1) - \sin(y(k))$$

Taking Z-transform of both side, we have

$$U(z) = Z\{y(k+2) - cy(k+1) - \sin(y(k))\}$$

In order to obtain perfect tracking, the system must have a stable inverse. Since there is no zero in this system, the inverse system is stable. Thus perfect tracking can be obtained not dependable on the value of c. Also, one can check that if the output to be track is stable (bounded), i.e., $|y^*(k)| \leq \alpha < \infty$, then the input is also bounded, as follows:

$$\begin{aligned}
u(k) &= y^*(k+2) - cy^*(k+1) - \sin(y^*(k)) \\
|u(k)| &= |y^*(k+2) - cy^*(k+1) - \sin(y^*(k))| \\
&\leq |y^*(k+2)| + |cy^*(k+1)| + |\sin(y^*(k))| \\
&\leq \alpha + c\alpha + \sin\alpha \\
&\leq \beta < \infty
\end{aligned}$$