EE3304 Lab 2: An Introduction to Computer Aided Digital Controller Design

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Objectives:

- Design and analysis of a continuous time system
- Design and analysis of a discrete-time system with digital PD
- Design and analysis of a discrete-time system with compensator

Discussions:

3. Design and Analysis of a Continuous Time System

Step 1: Open loop unit step response in Simulink

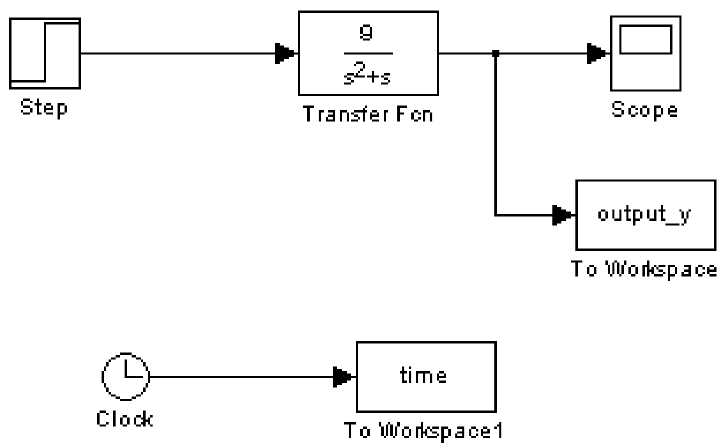

Figure 1: Function Blocks of Open Loop System

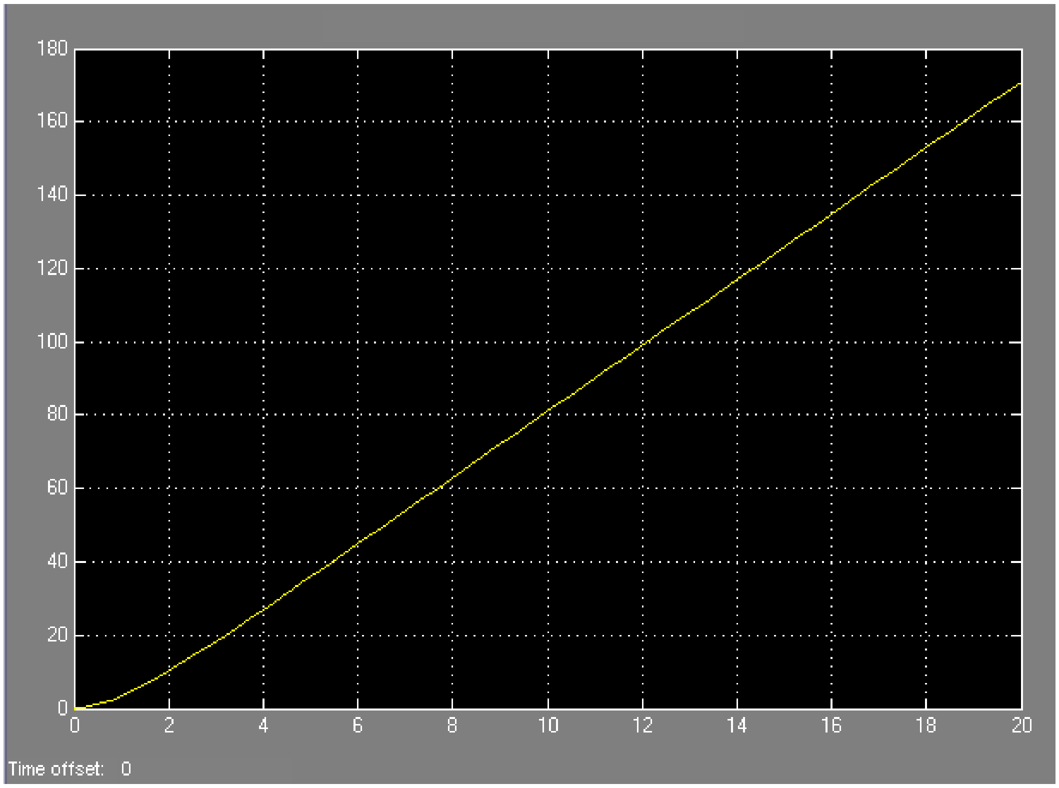

Figure 2: Step Response of Open Loop System

The unit step response of this open loop system ($G(s)$) shows an unbounded output. The output of the system $G(s)$ grows approximately linear with the time scales. The behavior of this unit step response can be mathematically proven by the following calculation:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{9}{s(s+1)} \qquad , \qquad U(s) = \frac{1}{s}$$

$$\begin{aligned} Y(s) &= \frac{9}{s(s+1)}\left(\frac{1}{s}\right) \\ &= \frac{9}{s^2(s+1)} \\ &= \frac{9}{s+1} + \frac{9}{s} - \frac{9}{s^2} \end{aligned}$$

$$\begin{aligned} y(t) &= L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{9}{s+1} - \frac{9}{s} + \frac{9}{s^2}\right\} \\ &= 9e^{-t} - 9 + 9t \end{aligned}$$

As time increase to infinity, the term $9t$ will dominate the system and hence the system's output is approximate linear relation with time. The $9e^{-t}$ term contributed to the exponential behavior at the start of the response but it die up very soon as t increase.

Step 2: Closed loop with a PD control under unity negative feedback

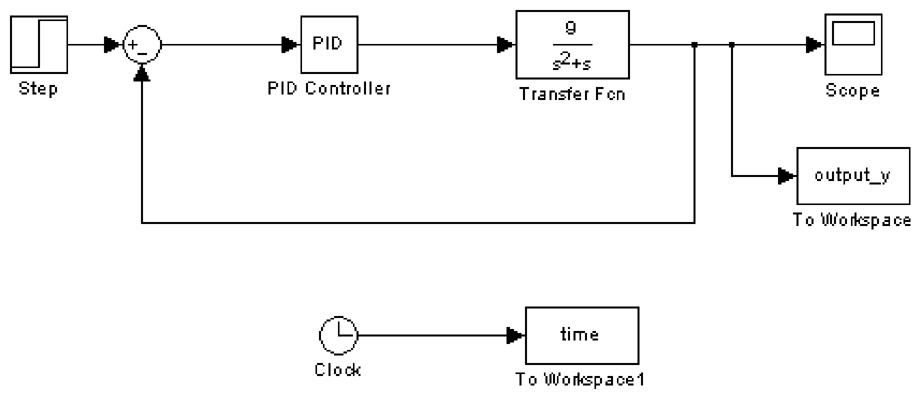

Figure 3: Function Blocks for Closed Loop System with PD Controller

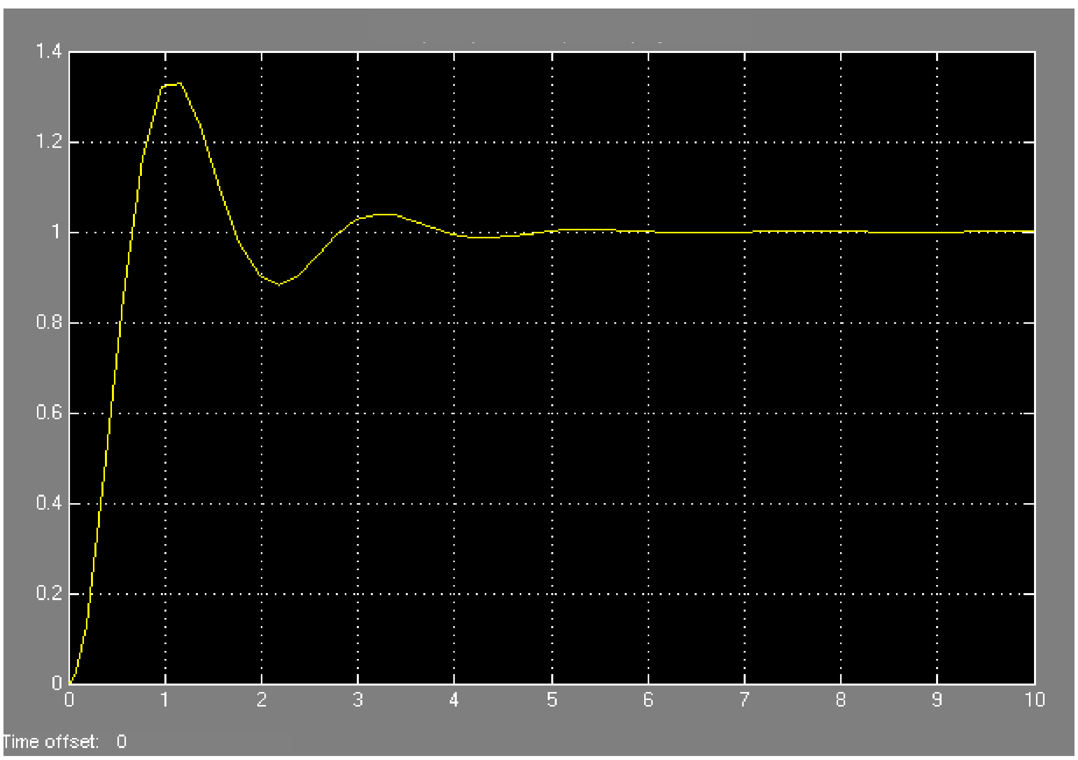

Figure 4: Step Response of Close Loop System with k_d=0.1

The output oscillates before it reaches the steady state at about 6 seconds. The maximum overshoot of the output is around 35%. Hence this is an under damping system.

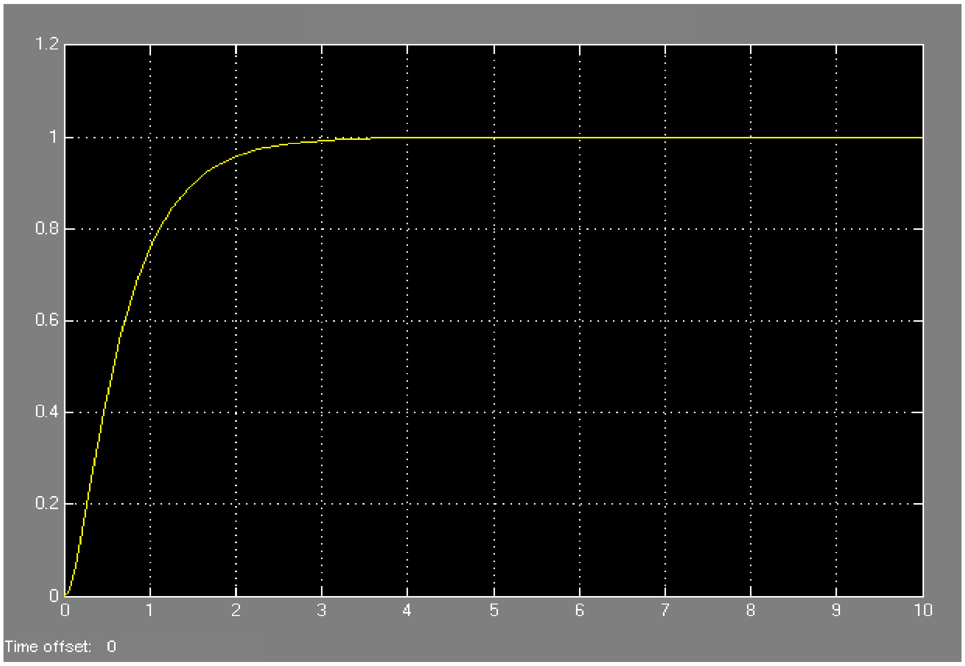

Figure 5: Step Response of Close Loop System with k_d=0.6

The system reaches steady state at around 3.5 seconds without any oscillating. This can be classified as critical damping system since the transient response is fast and non-oscillating.

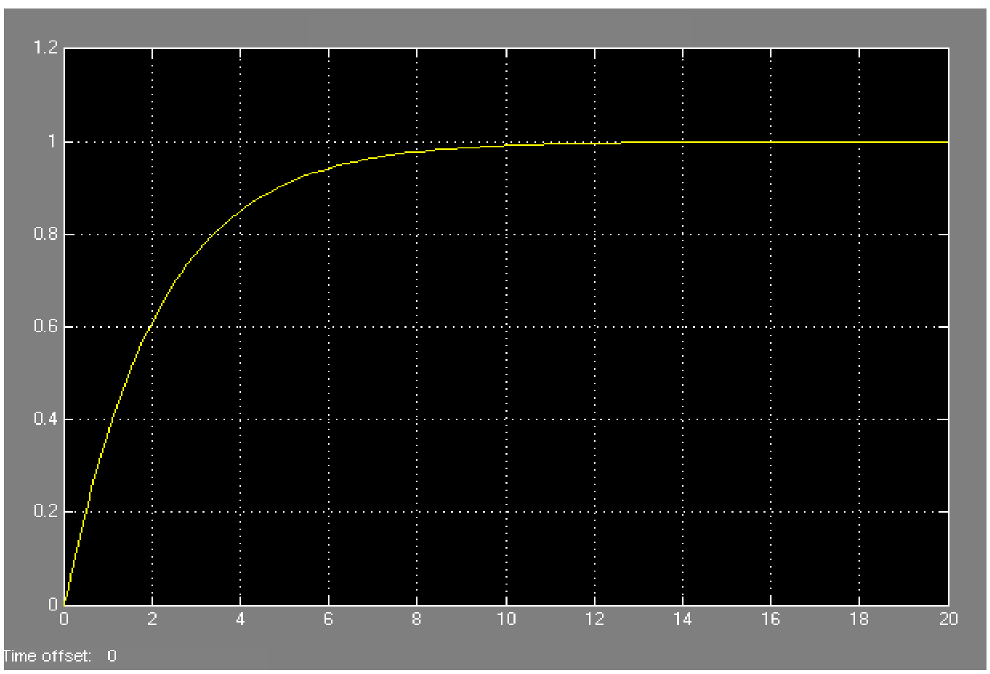

Figure 6: Step Response of Close Loop System with k_d=2

The system reaches steady state at around 11 seconds without any oscillating. Since the rise time is relatively slower than the previous response, this can be considered as an over damping system.

For $k_d = 0.6$, the closed-loop transfer function is given by

$$\frac{(1 + 0.6s)\left(\frac{9}{s^2 + s}\right)}{1 + (1 + 0.6s)\left(\frac{9}{s^2 + s}\right)} = \frac{5.4s + 9}{s^2 + 6.4s + 9}$$

Compare the closed-loop characteristic equation with a general 2nd order equation,

$$C_{cl} = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$6.4 = 2\zeta\omega_n\ ,\ \ 9 = \omega_n^2$$

$$\omega_n = 3$$
$$\zeta = 1.067$$

Since $\zeta \approx 1$, the closed-loop performance is analogous to a critical damped system.

The system bandwidth can be calculated using the following Matlab code:

```
% this program calculate
% bandwidth of the system
% in EE3304 lab2 step2
num = [5.4 9];
den = [1 6.4 9];
H = tf(num,den);
bandwidth(H)
```

The bandwidth calculated by Matlab is 6.1099 rad/s. For emulation to work in the following parts of the lab, the sampling rate should be at least 30 times faster than the system bandwidth, which is $6.1099 \times 30 = 183.3\ \text{rad/s} \approx 30$ Hz.

4. Design and Analysis of a Discrete-time System with Digital PD

Step 3: Discretize the analog PD controller using the backward rule

For $T = 0.03s$,

Using the backward rule: $s = \frac{z-1}{Tz}$

$$k_p + k_d s = 1 + 0.6s = 1 + 0.6\left(\frac{z-1}{0.03z}\right)$$
$$= \frac{0.63z - 0.6}{0.03z}$$

Set up the block diagram using Simulink with this value,

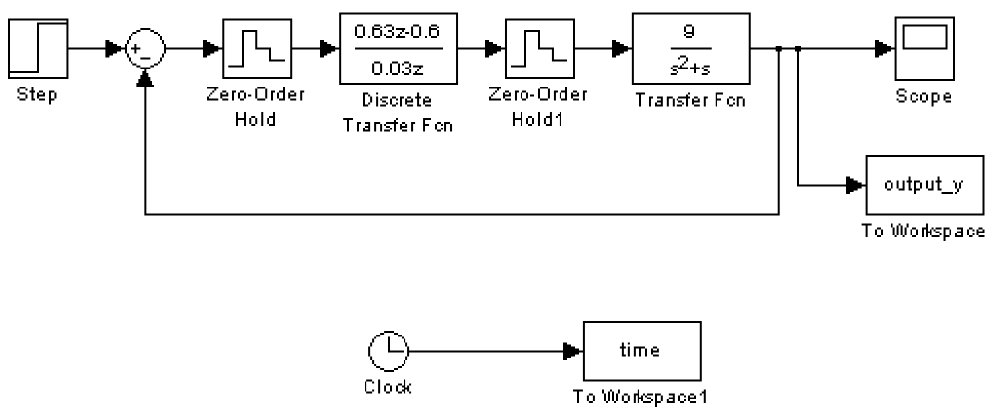

Figure 7: Function Blocks of Discretized Closed-loop System with T=0.03s

We can obtain the following plot:

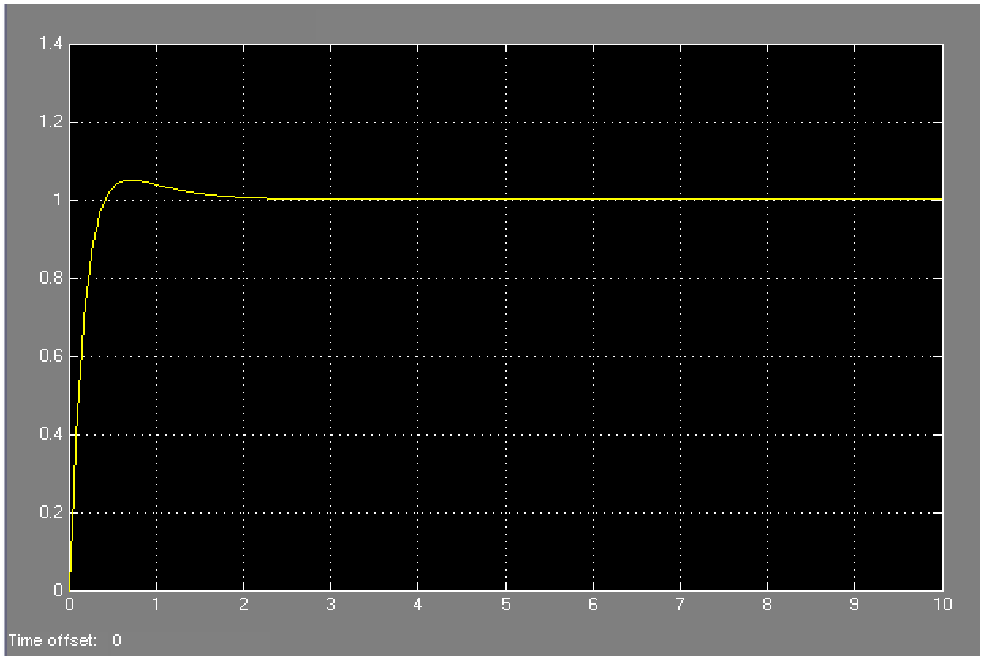

Figure 8: Step Response of Discretized PD Controller with T = 0.03s

The response shows an overshoot of approximate 5% without any oscillation. Hence this system can be considered as slightly under damping system. It reaches steady state value in 3 seconds.

For $T = 0.15s$,

Using the backward rule: $s = \frac{z-1}{Tz}$

$$k_p + k_d s = 1 + 0.6s = 1 + 0.6\left(\frac{z-1}{0.15z}\right)$$
$$= \frac{0.75z - 0.6}{0.15z}$$

Set up the block diagram using Simulink with this value,

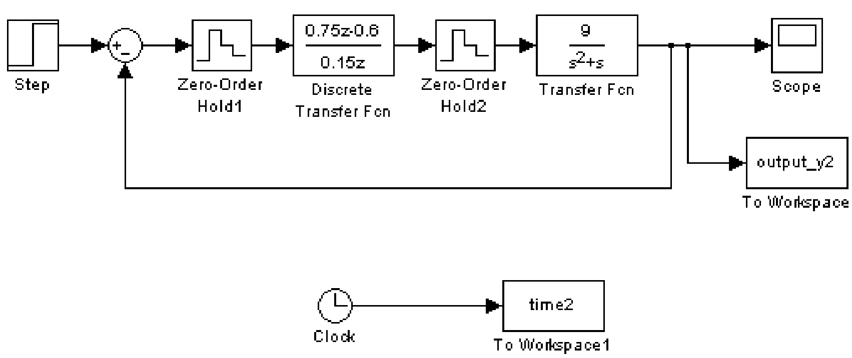

Figure 9: Function Blocks for Discretized Close-loop System with T=0.15s

We can obtain the following plot:

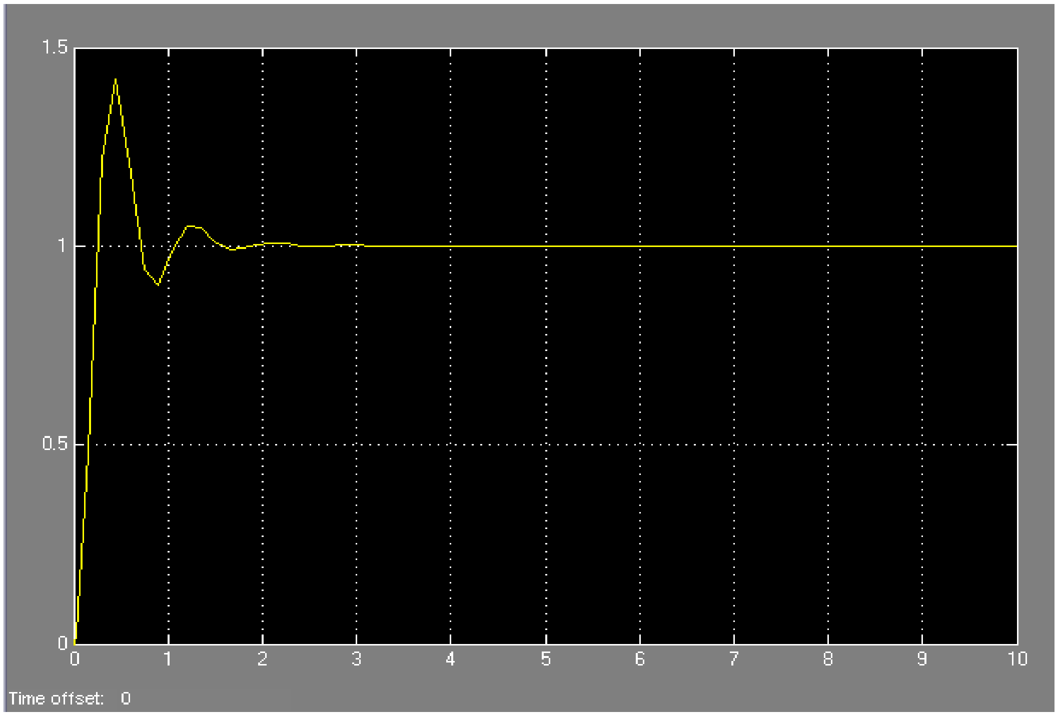

Figure 10: Step Response of Discretized Close-loop System with T = 0.15s

The step response shows an overshoot of about 40% and some oscillations occurs before the output reaches its steady state value. This system can be considered as under damping system. It reaches the steady state at about 3 seconds.

Compare the discrete step response with the original continuous step response, we obtain the following graph:

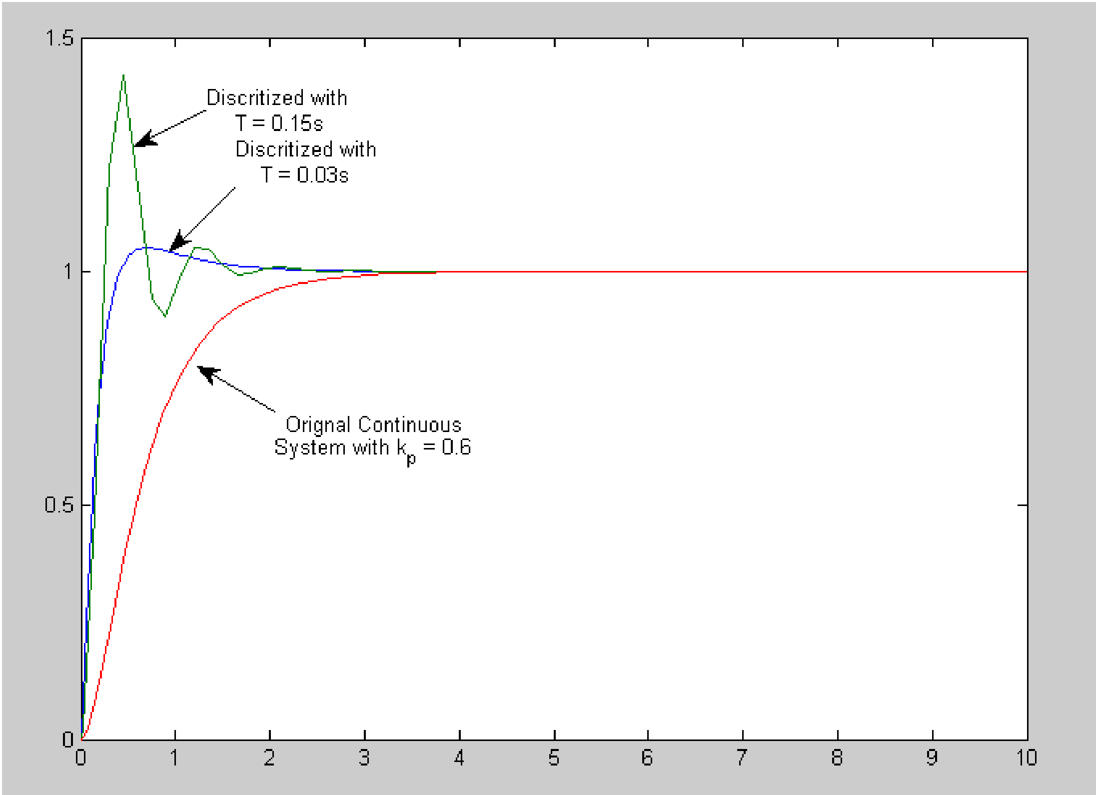

Figure 11: Step Response of Continuous System and its Discretized Response.

We observe that for the same value of K_d (0.6), the discrete equivalent system will have a response of more oscillation and behave like an under damping system.

When $T = 0.03s$, it is approximately 30 times faster than the system bandwidth as calculated earlier. Hence the closed-loop response of this system will be better than that when $T = 0.15s$ which is just approximately 6 times faster than the original system bandwidth. At the lower sampling frequency, a less desired oscillatory transient response will be obtained. Hence for the cases where the bandwidth is too low, additional modification is needed to achieve a better response.

In all for this case, both the discrete system of sampling time $T = 0.03s$ and $T = 0.15s$ has stable step response.